

Kinetic exchange models for social opinion formation

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(Dated: July 5, 2010)

We propose a minimal model for the collective dynamics of opinion formation in the society, by modifying kinetic exchange dynamics studied in the context of income, money or wealth distributions in a society. This model has an intriguing spontaneous symmetry breaking transition.

PACS numbers: 87.23.Ge 02.50.-r

Keywords: Econophysics; Sociophysics; kinetic theory

I. INTRODUCTION

A very interesting problem in studying society and social dynamics is the one of “opinion formation”, which is a collective dynamical phenomenon, and as such are closely related to problems of competing cultures or languages [1–3]. It deals with a “measurable” response of the society to e.g., political issues, acceptances of innovations, etc. A number of models of competing opinions have been introduced to study it, e.g., the “voter” model (which has a binary opinion variable with the opinion alignment proceeding by a random choice of neighbors) [4], or the Sznajd-Weron discrete opinion formation model (where more than just a pair of spins is associated with the decision making procedure) [5]. There have been studies of systems with more than just two possible opinions [6], or where the opinion of individuals is represented by a “continuous” variable [7–9]. Since opinion formation in a human society is mediated by social interactions between individuals, such social dynamics was considered to take place on a network of relationships by Holme and Newman [10]. Several other significant studies have followed, which we not mention here.

A two body exchange dynamics has already been developed in the context of modelling income, money or wealth distributions in a society [11–14]. Detailed analytical structure of the collective dynamics in these models are now considerably well-developed [15, 16]. Here,

we propose a minimally modified version of those models for the collective dynamics of opinion formation in the society.

II. KINETIC EXCHANGE MODELS OF MARKET

Recently physicists and mathematicians have been interested in studying the wealth distributions in a closed economy using kinetic exchange mechanism, which has led to new insights into this field (see Refs. [11–14]). The general aim was to study a many-agent statistical model of closed economy (analogous to the kinetic theory model of ideal gases) [17–19], where N agents exchange a quantity x , that may be defined as wealth. The states of agents are characterized by the wealth $\{x_i\}$, $i = 1, 2, \dots, N$, and the total wealth $W = \sum_i x_i$ is conserved. The question of interest is: “What is the equilibrium distribution of wealth $f(x)$, such that $f(x)dx$ is the probability that in the steady state of the system, a randomly chosen agent will be found to have wealth between x and $x + dx$?”

The evolution of the system is carried out according to a prescription, which defines the trading rule between agents, where the agents interact with each other through a pair-wise interaction characterized by a saving parameter λ , with $0 \leq \lambda \leq 1$. The dynamics of the model (CC) is as follows [19]:

$$\begin{aligned}x'_i &= \lambda x_i + \epsilon(1 - \lambda)(x_i + x_j), \\x'_j &= \lambda x_j + (1 - \epsilon)(1 - \lambda)(x_i + x_j).\end{aligned}\quad (1)$$

It can be noticed that in this way, the quantity x is con-

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served during the single transactions: $x'_i + x'_j = x_i + x_j$, where x'_i and x'_j are the agent wealths after the transaction has taken place.

This model for $\lambda > 0$ leads to an equilibrium distribution, with a mode $x_m > 0$ and a zero limit for small x . For $\lambda = 0$, the model reproduces the results of Yakovenko [18], where the equilibrium distribution is the Gibb's distribution. In general, the functional form for such distributions was conjectured to be a Γ -distribution on the basis of an analogy with the kinetic theory of gases:

$$f(x) = \frac{1}{\Gamma(n)} \left(\frac{n}{\langle x \rangle} \right)^n x^{n-1} \exp \left(-\frac{nx}{\langle x \rangle} \right), \quad (2)$$

where

$$n = \frac{D(\lambda)}{2} = 1 + \frac{3\lambda}{1-\lambda}. \quad (3)$$

Indeed, starting from the Maxwell-Boltzmann distribution for the particle velocity in a D dimensional gas, it can be shown that the equilibrium kinetic energy distribution coincides with the Gamma-distribution (2) with $n = \frac{D}{2}$. This conjecture is remarkably consistent with the fitting provided to numerical data [20, 21].

As a further generalization, the agents could be assigned different saving propensities λ_i [22]. In particular, uniformly distributed λ_i in the interval $[0, 1)$ had been studied numerically in Refs. [22]. This model (CCM) is described by the trading rule

$$\begin{aligned} x'_i &= \lambda_i x_i + \epsilon[(1 - \lambda_i)x_i + (1 - \lambda_j)x_j], \\ x'_j &= \lambda_j x_j + (1 - \epsilon)[(1 - \lambda_i)x_i + (1 - \lambda_j)x_j]. \end{aligned} \quad (4)$$

One of the main features of this model, which is supported by theoretical considerations [15, 23, 24], is that the wealth distribution exhibits a robust power-law at large values of x ,

$$f(x) \propto x^{-\alpha-1}, \quad (5)$$

with a Pareto exponent $\alpha = 1$ largely independent of the details of the λ -distribution. Note that other values of exponents can also be generated by modifying the exchange rules [12].

III. A KINETIC EXCHANGE MODEL FOR OPINION FORMATION

Toscani [25] had recently introduced and discussed kinetic models of (continuous) opinion formation involving both exchange of opinion between individual agents and diffusion of information. He showed that there are conditions which ensure that the kinetic model reaches non-trivial stationary states in case of lack of diffusion in correspondence of some opinion point, and obtained analytical results by considering a suitable asymptotic limit of the model yielding a Fokker-Planck equation for the distribution of opinion among individuals. Based on this

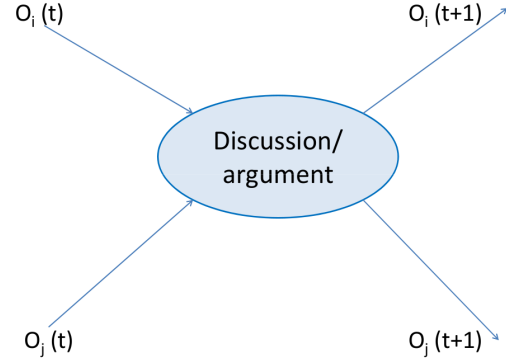


FIG. 1. Schematic diagram of the minimal model where random discussions/arguments between two persons i and j with opinions $O_i(t)$ and $O_j(t)$, respectively, cause the update of opinions $O_i(t+1)$ and $O_j(t+1)$.

model, During et al [26] proposed another mathematical model for opinion formation in a society that is built of two groups, one group of ordinary people and one group of strong opinion leaders. Starting from microscopic interactions among individuals, they arrived at a macroscopic description of the opinion formation process that is characterized by a system of FokkerPlanck-type equations. They discussed the steady states of the system, and extended it to incorporate emergence and decline of opinion leaders. On a different approach, Iniguez et al [27] examined a situation in which these non-identical individuals form their opinions in information-transferring interactions with others. They developed a dynamic network model, where they consider short range interactions for direct discussions between pairs of individuals, long range interactions for sensing the overall opinion modulated by the attitude of an individual, and external field for outside influence.

Following the CC and CCM models, described in the earlier section, we now propose a minimal model for the collective dynamics of opinion $O_i(t)$ of the i -th person in the society of N ($N \rightarrow \infty$) persons:

$$\begin{aligned} O_i(t+1) &= \lambda_i O_i(t) + \epsilon \lambda_j O_j(t), \\ O_j(t+1) &= \lambda_j O_j(t) + \epsilon' \lambda_i O_i(t), \end{aligned} \quad (6)$$

where $-1 \leq O_i(t) \leq 1$ for all i and t , and $0 \leq \lambda_i \leq 1$'s are *quenched* variables (do not change with time, but vary from person to person), and ϵ and ϵ' are *annealed* variables (change with time), that are random numbers uniformly distributed between 0 and 1.

The above described model dynamics, follows the two-body “discussions/arguments” modelled as *scattering processes* and depicted schematically in Fig. 1. It is based on the logic that during the discussion/argument

event with any person j , the person i with high/low “conviction” (parametrized by the λ_i), will *retain* his/her own earlier opinion $O_i(t)$ proportional to the factor λ_i , and be *influenced* to change the opinion by the j -th person’s influence determined by a contribution which will depend on the j -th person’s conviction λ_j (and not by the factor $1 - \lambda_j$ as in market dynamics Eq. 1 or 4). Also, as *no conservation in opinion is possible* (unlike in the market models above), the annealed variables ϵ and ϵ' are now considered to be *uncorrelated*. Additionally we assume that $|O_i(t)| \leq 1$, for all i and t .

A. Homogeneous conviction factor case

When we assume $\lambda_i = \lambda$ for all i (equivalent to the CC model for market dynamics), the above equations reduce to

$$\begin{aligned} O_i(t+1) &= \lambda(O_i(t) + \epsilon O_j(t)), \\ O_j(t+1) &= \lambda(O_j(t) + \epsilon' O_i(t)). \end{aligned} \quad (7)$$

This leads to an intriguing *spontaneous symmetry breaking* transition beyond a threshold value of $\lambda_c = 2/3$. Specifically, following the above dynamics, starting from random (drawn uniformly) positive and negative values of $O_i(0)$ (at $t = 0$) (“symmetric” state, when the *order parameter* $\langle O \rangle \equiv (1/N) \sum_i O_i(t = 0) = 0$), leads the system to collectively evolving to two kinds of state:

- (i) “Para” or “indifferent” state, where $O_i(t)$ ’s are *all* zeros ($\langle O \rangle = 0$) after a “relaxation” time τ , for λ values less than $\lambda_c = 2/3$; or
- (ii) “Symmetry broken” or “polarised” state, where $O_i(t)$ ’s are either all positive or all negative ($\langle O \rangle \neq 0$) after a “relaxation” time τ , for $\lambda > 2/3$.

One can easily see that for λ values less than $2/3$, with $\langle \epsilon \rangle = 1/2$, the recursion relation for the *order parameter* $\langle O \rangle$ becomes a simple multiplier equation with the value of the multiplier less than unity, leading to $\langle O \rangle = 0$ eventually. For higher values of λ , the fluctuations in ϵ are important (and cannot be replaced by its simple average, as above) because of asymmetric contributions from the second term of both the above equations (if the contribution of the second term in Eq. 7 takes the value of $|O(t+1)|$ to greater than unity, only partial contribution of the second term is accepted, while for its lower values the acceptance is full). We find, this seemingly leads to a *discontinuous* or “first order” symmetry breaking transition at $\lambda_c = 2/3$ (see Fig. 2). The details of this transition will be reported elsewhere [28].

B. Heterogeneous conviction factor case

Here, we assume λ_i ’s to be uniformly spread in the interval $[0,1]$ (equivalent to the CCM model for market

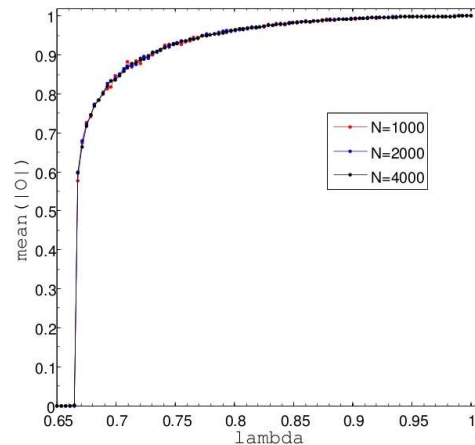


FIG. 2. The variation of the order parameter $\langle O \rangle \equiv (1/N) \sum_i O_i(t)$ against λ .

dynamics). We study similarly, starting from “symmetric” states (with random positive and negative values of $O_i(0)$, the evolution of the system. The dynamics here leads collectively to the “Polarized” or “Symmetry broken” state ($O_i(t)$ are either all positive or all negative, for all i , and times $t > \tau$) only. The “indifferent” states (with $O_i(t) = 0$ for all i , for times $t > \tau$) disappear in the large system size limit, although this is clearly a *fixed point* of the dynamics given by Eq. 6. We believe, this is also a clear feature of the opinion dynamics model proposed by Iniguez et al [27], where also this state is surely a fixed point of their model.

It may be noted that the above dynamics can be considerably modified by the presence of “polarizing field” terms h_i (fixed over time t but dependent on person i), added linearly to the dynamical equations Eq. 6 of $O_i(t)$. Such “fields” can be provided by the “influences” of the media in the society. Detailed analyses of the field terms, etc. will be reported elsewhere [29].

IV. DISCUSSION AND SUMMARY

The appearance of spontaneous symmetry breaking in this kinetic opinion exchange model is truly remarkable. It appears to be one of the simplest collective dynamical model of many-body dynamics showing non-trivial phase transition behaviour. The details of this transition is under investigation and will be reported elsewhere.

ACKNOWLEDGMENTS

The authors acknowledge F. Abergel, A.S. Chakrabarti, A. Chatterjee, A. Jedidi, K. Kaski and N. Millot for useful discussions and comments.

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